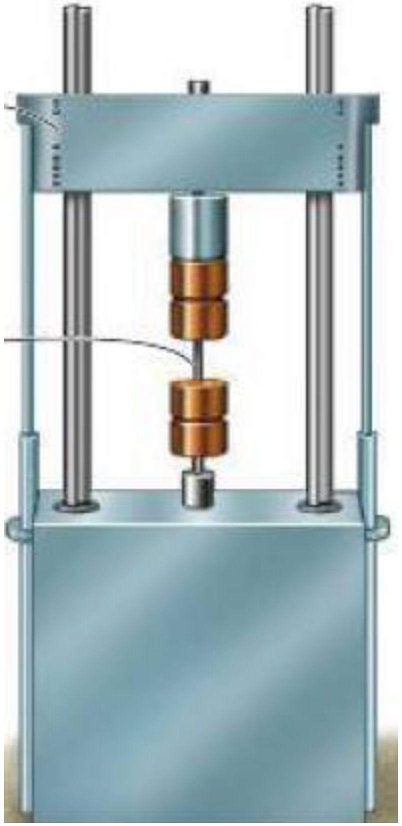
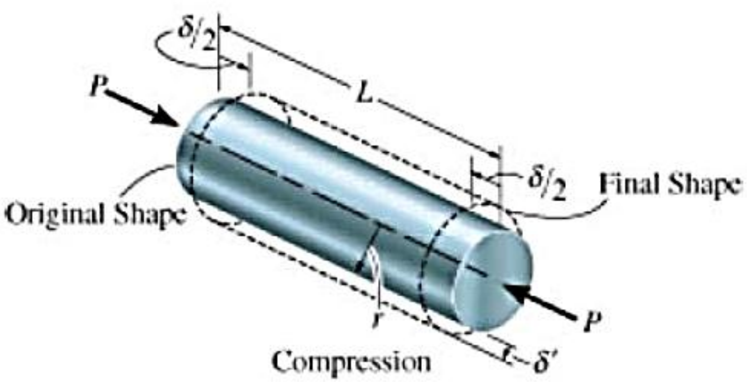
# 3 Mechanical Properties of Materials 83

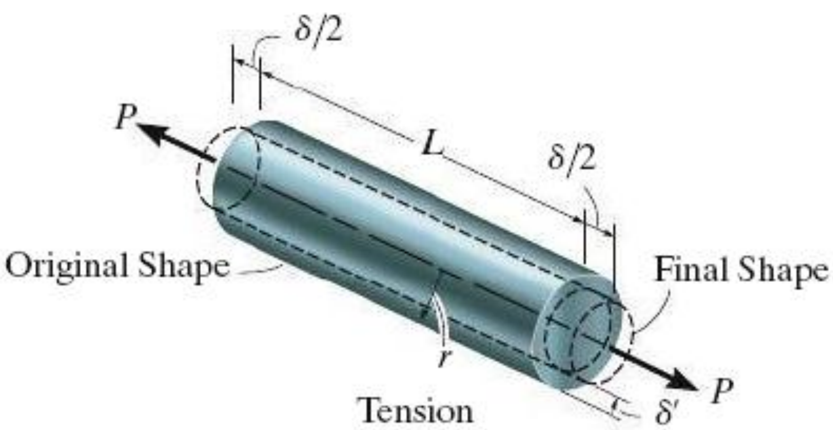




## 3.6 Poisson's Ratio

- When body subjected to axial tensile force, it elongates and **contracts** laterally
- Similarly, it will contract and its sides expand laterally when subjected to an axial **compressive force**



- Strains of the bar are:

$$\varepsilon_{long} = \frac{\delta}{L} \qquad \varepsilon_{lat} = \frac{\delta'}{r}$$

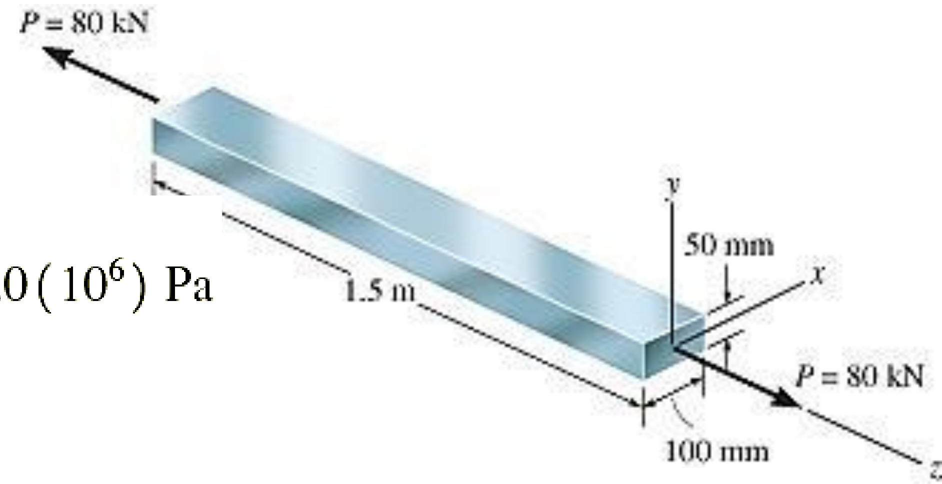
- Early 1800s, S.D. Poisson realized that within elastic range, the ration of the two strains is a constant value, since both $\delta$ *and* $\delta'$ are proportional.

$$\textit{Poisson's ratio, } \nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$$

**ν** is unique for homogenous and isotropic material

- Why negative sign? Longitudinal elongation cause lateral contraction (-ve strain) and vice versa
- Lateral strain is the same in all lateral (radial) directions
- Note that, no force or stress acts in a lateral direction in order to strain the material; the strain is caused only by axial force.
- Poisson's ratio is dimensionless, $0 \leq \nu \leq 0.5$

Ex:- Bar is made of **A-36 steel** and behaves elastically. Determine change in its length and change in dimensions of its cross section after load is applied.




## SOLUTION

The normal stress in the bar is

$$\sigma_z = \frac{P}{A} = \frac{80(10^3)\ \text{N}}{(0.1\ \text{m})(0.05\ \text{m})} = 16.0(10^6)\ \text{Pa}$$

From the table on the inside back cover for A-36 steel $E_{st} = 200$ GPa, and so the strain in the $z$ direction is

$$\epsilon_z = \frac{\sigma_z}{E_{st}} = \frac{16.0(10^6)\ \text{Pa}}{200(10^9)\ \text{Pa}} = 80(10^{-6})\ \text{mm/mm}$$

The axial elongation of the bar is therefore

$$\delta_z = \epsilon_z L_z = [80(10^{-6})](1.5\ \text{m}) = 120\ \mu\text{m}$$

Using Eq. 3–9, where $\nu_{st} = 0.32$ as found from the inside back cover, the lateral contraction strains in *both* the $x$ and $y$ directions are

$$\epsilon_x = \epsilon_y = -\nu_{st}\,\epsilon_z = -0.32[80(10^{-6})] = -25.6\ \mu\text{m/m}$$

Thus the changes in the dimensions of the cross section are

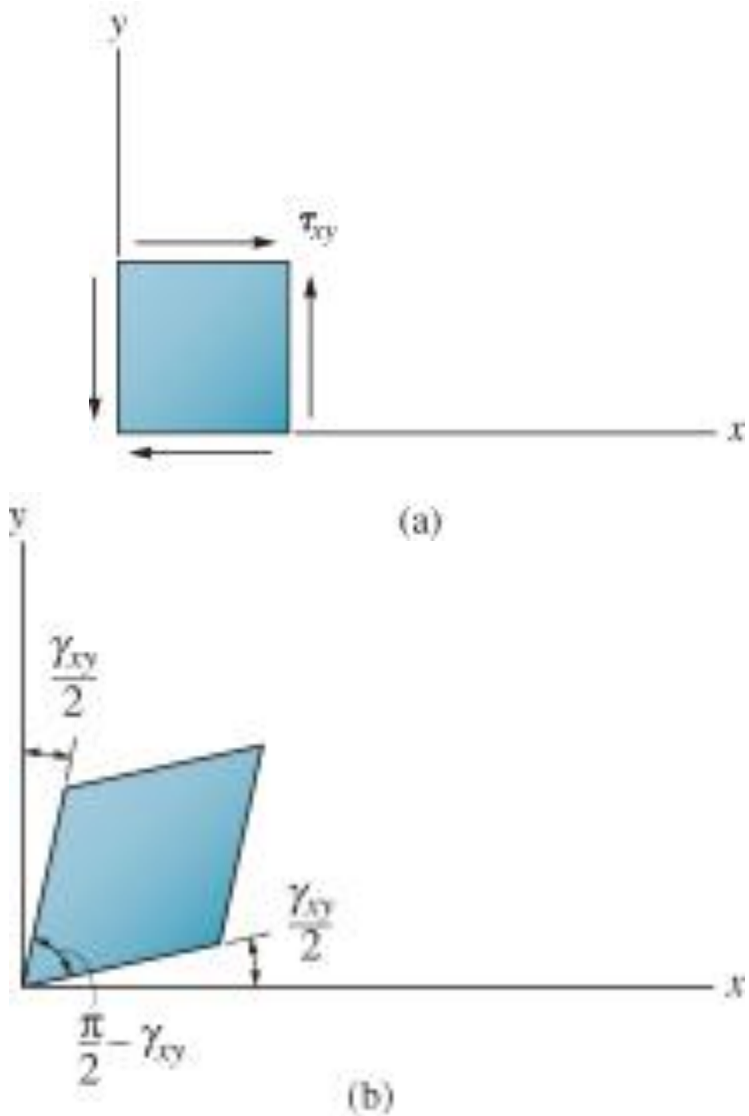
$$\delta_x = \epsilon_x L_x = -[25.6(10^{-6})](0.1\ \text{m}) = -2.56\ \mu\text{m} \qquad \textit{Ans.}$$

$$\delta_y = \epsilon_y L_y = -[25.6(10^{-6})](0.05\ \text{m}) = -1.28\ \mu\text{m} \qquad \textit{Ans.}$$

## 3.7 The Shear Stress–Strain Diagram

- Use thin-tube specimens and subject it to torsional loading

- If the material is homogeneous and isotropic, then the shear stress as a result of torsional loading will caused uniform distortion.
- Record measurements of applied torque and resulting angle of twist to construct shear stress and shear strain diagram.



# MECHANICAL PROPERTIES OF MATERIALS



- Material will exhibit linear-elastic behavior till its proportional limit, $\tau_{pl}$
- Strain-hardening continues till it reaches ultimate shear stress, $\tau_u$
- Material loses shear strength till it fractures, at stress of $\tau_f$

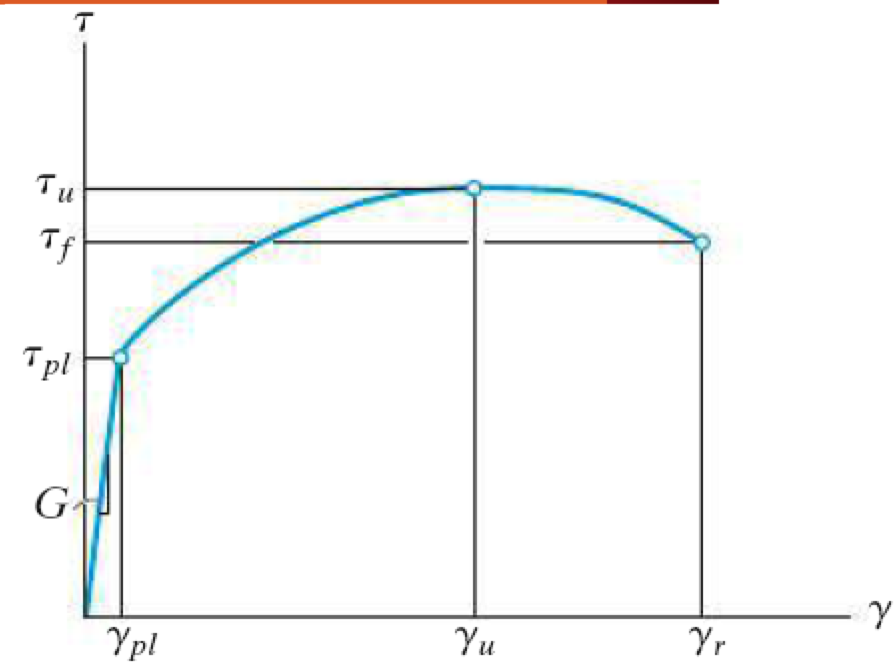



**Fig. 3–24**

- Hooke's law for shear $\boldsymbol{\tau = G\gamma}$

***G*** is shear modulus of elasticity or modulus of rigidity

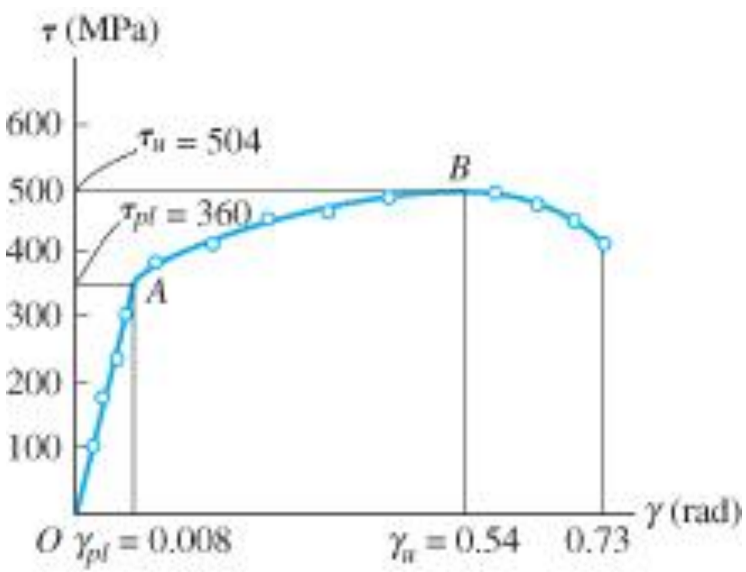
***G*** can be measured as slope of line on τ-γ diagram, $\boldsymbol{G = \tau_{pl} / \gamma_{pl}}$

The three material constants *E*, *ν*, and *G* is related by

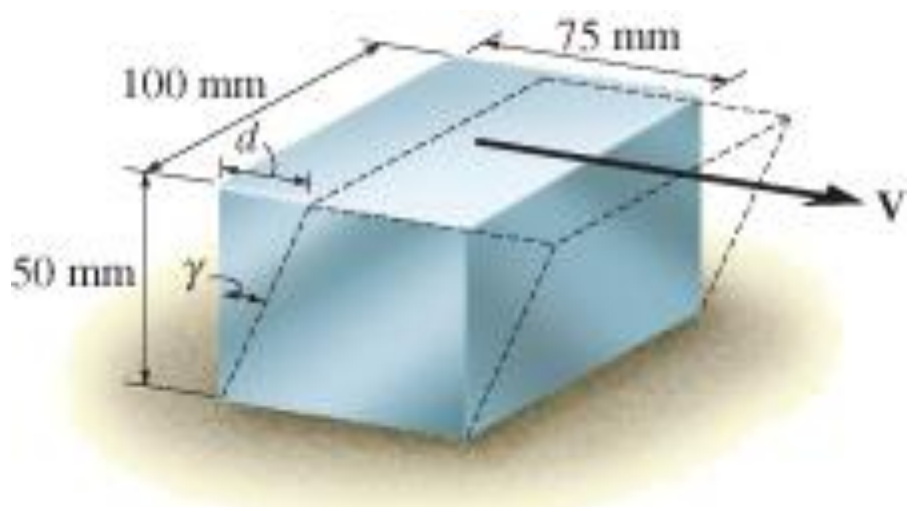
$$G = \frac{E}{2(1 + \nu)}$$

**EX:-** Specimen of titanium alloy tested in torsion & shear stress-strain diagram shown below.

1. Determine shear modulus ***G***, proportional limit, and ultimate shear stress.
2. Also, determine the maximum distance ***d*** that the top of the block shown, could be displaced horizontally if material behaves elastically when acted upon by **V**. Find magnitude of **V** necessary to cause this displacement.




(a)



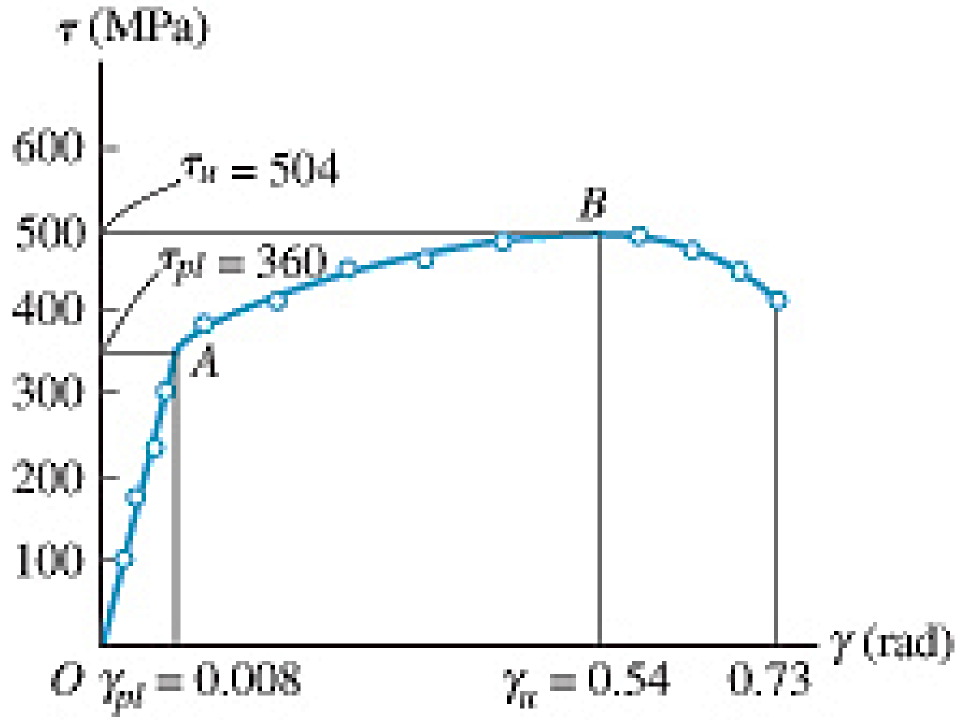

(b)

## Shear modulus

Obtained from the slope of the straight-line portion *OA* of the τ-γ diagram. Coordinates of *A* are (0.008 rad, 360 MPa)

$$G = \frac{360 \text{ MPa}}{0.008 \text{ rad}}$$

$$= 45(10^3) \text{ MPa}$$




(a)

## Proportional limit

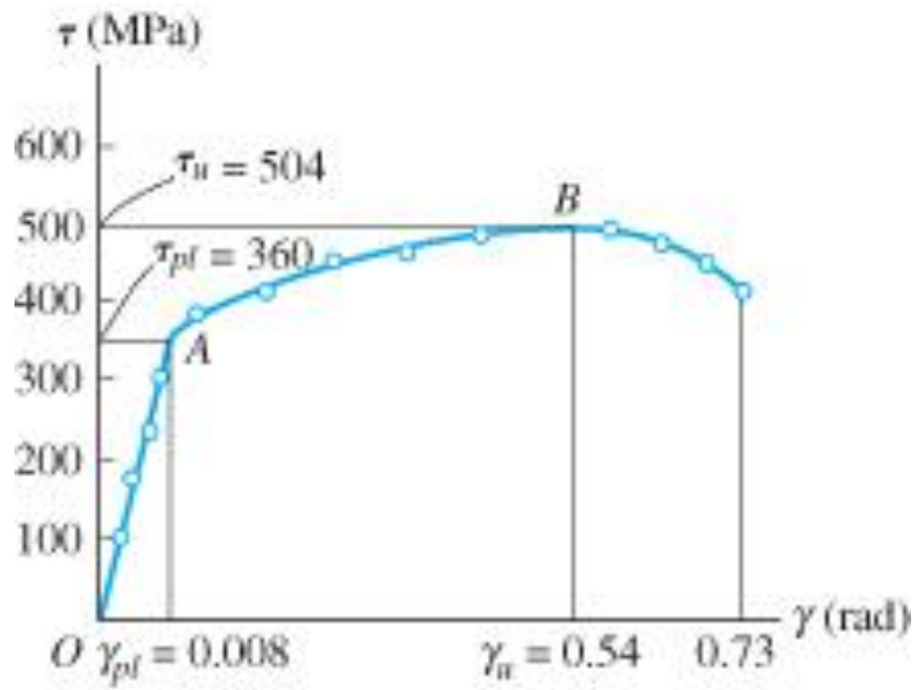
By inspection, graph ceases to be linear at point A, thus,

$\tau_{pl} = 360$ MPa

## Ultimate stress

From graph,

$\tau_u = 504$ MPa

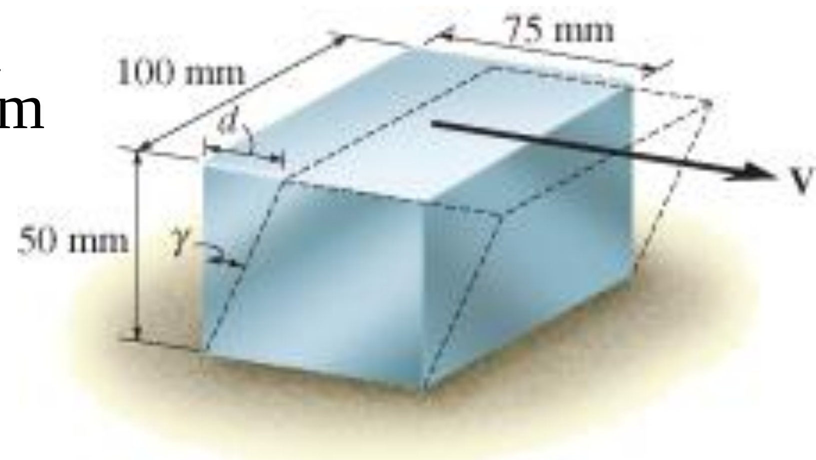



(a)

Maximum elastic displacement and shear force By inspection, graph ceases to be linear at point A, thus,

$$\tan(0.008\text{ rad}) \approx 0.008\text{ rad} = \frac{d}{50\text{ mm}}$$

$$d = 0.4\text{ mm}$$



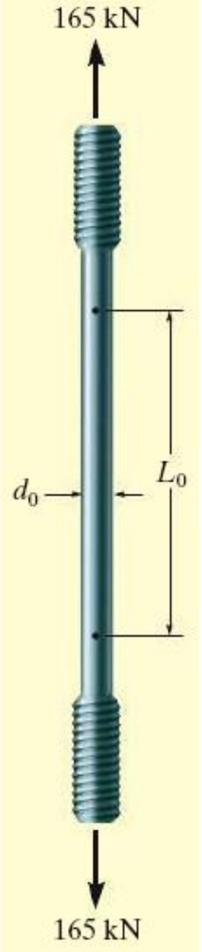
(b)

$$\tau_{avg} = \frac{V}{A}$$

$$360\text{ MPa} = \frac{V}{(75\text{ mm})(100\text{ mm})}$$

$$V = 2700\text{ kN}$$

**EX:-** An aluminum specimen shown in Fig. 3–26 has a diameter of $d_0 = 25$ mm and a gauge length of $L_0 = 250$ mm. If a force of 165 kN elongates the gauge length 1.20 mm, determine the modulus of elasticity. Also, determine by how much the force causes the diameter of the specimen to contract. Take $G_{al} = 26$ GPa and $\sigma_Y = 440$ MPa.




SOLUTION

**Modulus of Elasticity.** The average normal stress in the specimen is

$$\sigma = \frac{P}{A} = \frac{165(10^3)\text{ N}}{(\pi/4)(0.025\text{ m})^2} = 336.1\text{ MPa}$$

and the average normal strain is

$$\epsilon = \frac{\delta}{L} = \frac{1.20\text{ mm}}{250\text{ mm}} = 0.00480\text{ mm/mm}$$

Since $\sigma < \sigma_Y = 440$ MPa, the material behaves elastically. The modulus of elasticity is therefore

$$E_{al} = \frac{\sigma}{\epsilon} = \frac{336.1(10^6)\text{ Pa}}{0.00480} = 70.0\text{ GPa} \qquad \textit{Ans.}$$

**Contraction of Diameter.** First we will determine Poisson's ratio for the material using Eq. 3–11.

$$G = \frac{E}{2(1 + \nu)}$$

$$26\text{ GPa} = \frac{70.0\text{ GPa}}{2(1 + \nu)}$$

$$\nu = 0.347$$

Since $\epsilon_{\text{long}} = 0.00480\text{ mm/mm}$, then by Eq. 3–9,

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$

$$0.347 = -\frac{\epsilon_{\text{lat}}}{0.00480\text{ mm/mm}}$$

$$\epsilon_{\text{lat}} = -0.00166\text{ mm/mm}$$

The contraction of the diameter is therefore

$$\delta' = (0.00166)(25\text{ mm})$$

$$= 0.0416\text{ mm} \qquad \textit{Ans.}$$

# * 3.8 Failure of Materials Due to Creep and Fatigue

**In some cases,**

- A member may have to be used in an environment for which loadings must be sustained over long periods of time at elevated temperatures, or in other cases, the loading may be repeated or cycled.
- We will not consider these effects in this book, although we will briefly mention how one determines a material's strength for these conditions, since they are given special treatment in design.

## Creep

- Occurs when material supports a load for very long period of time, and continues to deform until a sudden fracture or usefulness is impaired.

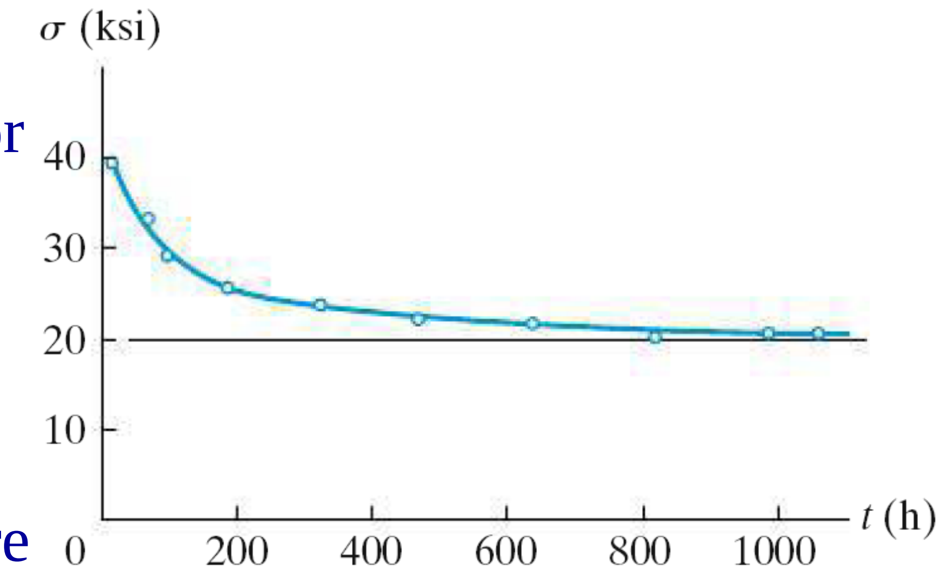
- Is only considered when metals and ceramics are used for structural members or mechanical parts subjected to high temperatures.

## Creep

- Other materials (such as polymers & composites) are also affected by creep without influence of temperature.
- Stress and/or temperature significantly affects the rate of creep of a material.
- Creep strength represents the ***highest initial*** *stress* the material can withstand during *given time* without causing *specified creep strain*

## ❖ Simple method to determine creep strength

- Test several specimens simultaneously
  - ✓ At constant temperature, but
  - ✓ Each specimen subjected to different axial stress
- Measure time taken to produce allowable strain or rupture strain for each specimen
- Plot stress vs. strain
- Creep strength inversely proportional to temperature and applied stresses




$\sigma$–$t$ diagram for stainless steel at 1200°F and creep strain at 1%

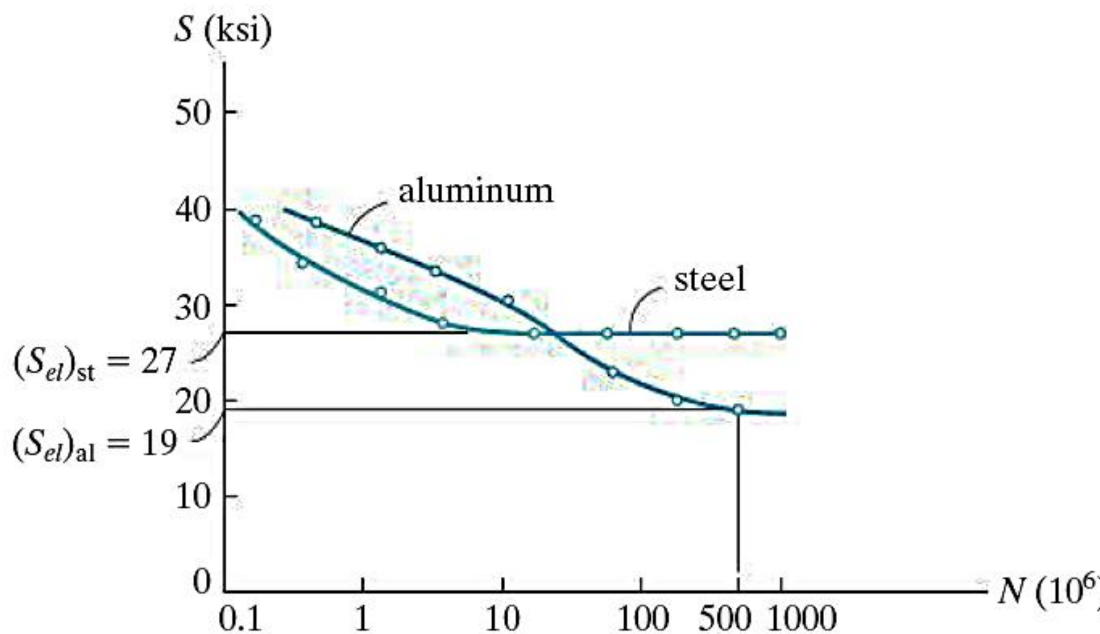
The long-tem application of the cable loading on this pole has caused the pole to deform due to creep.

# Fatigue

- Defined as gradual deterioration of a material that is subjected to time varying loads.
- Needs to be accounted for in design of connecting rods (e.g. steam/gas turbine blades, connections/supports for bridges, railroad wheels/axles and parts subjected to cyclic loading)
- Fatigue occurs at a stress *lesser* than the material's yield stress
- Also referred to as the endurance or fatigue limit

## ❖ Method to get value of fatigue

- Series of specimens are subjected to a specified stress and cycled to failure.
- Plot stress (S) against number of cycles-to-failure N (S-N diagram) on logarithmic scale – also known as stress-cycled diagram.




*S–N* diagram for steel and aluminum alloys
(*N* axis has a logarithmic scale)

# MECHANICAL PROPERTIES OF MATERIALS



The design of members used for amusement park rides requires careful consideration of cyclic loadings that can cause fatigue.

Engineers must account for possible fatigue failure of the moving parts of this oil-pumping rig.